

Areas to Review Before Taking the Math Section of
Highly Qualified Test for Paraprofessionals

Addition, subtraction, multiplication and division of fractions including unlike denominator and whole numbers.

- $2/4 + 3/8$
- $5 \frac{3}{4} - 2 \frac{1}{2}$
- $2 \frac{3}{4} \times 2/3$
- $\frac{3}{4} \div 5$

Addition, subtraction, multiplication and division of whole numbers using decimals

- $34.2 + 8 + 3.45$
- $34.9 - 7.23$
- 3.9×38
- $38.4 \div 7$

Percentages*

- If you have 50 cookies and eat 45, what percent of cookies are left?
- There are 36 presents under the tree. How many presents did mom and dad put under the tree if they put $\frac{1}{4}$?
- 15 % of what number is 45?
- Find $\frac{2}{3}$ of a given number.

Average: Know how to find the average for a set of numbers (3 numbers, 4 numbers, 5 number).

Add and subtract positive and negative numbers:

- $16 + (-9) =$
- $18 - (-5) =$

Solve for x

- If $x - 24 = 62$, then x is
- If $-3x = 36$, then x is

Simplify

- $-5b + 4a - 4a - 9b =$
- $12x + 5y + 7y - 3x =$

Factor a trinomial (Write an equivalent expression with two terms)

- $y^2 - 15y + 45$

Four Quadrant Grid Graph*

Locate points on a grid graph using coordinates with both positive and negative numbers
(-4, 4), (4 -4)

Find area and perimeter of a triangle and a rectangle*

Compare volume of liquids in different size containers by converting all containers to one unit of measure. If one pint has 16 ounces, which of the following will contain the most liquid if the container is full?

- 5 ½ pint cartons
- 1 ¼ quart vase
- ¾ gallon jug.

Graphs*: Be able to read and answering questions using information given on a line graph and a bar graph. Read questions carefully.

Solving problems using data (numbers) given in a chart format. Read questions carefully. Check your math.

Reading a map. Calculate how many miles from a beginning to an end point on a map. Several routes are given. You calculate which route is the shortest or longest.

Select the prime factorization for a given number*.

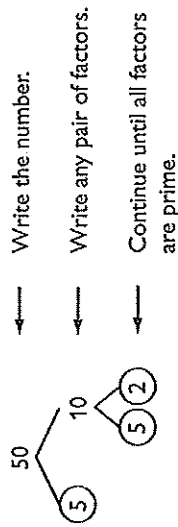
Bank Statement: Be able to read and answer questions about a bank statement. Know the meaning of terms likes checks, debits, deposits, previous balance, end of statement period, money withdrawn, etc. All answers are on the statement.

Using estimation or approximation to find the best answer to a given problem. Review how to round whole numbers up or down and how to estimate with a decimal.*

* indicates additional information as part of the handout

Prime Factorization

Every composite number can be expressed as a product of prime factors. Use a factor tree to find the prime factors.



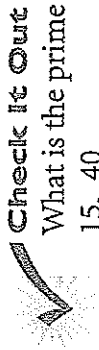
Write the number.

Write any pair of factors.

Continue until all factors are prime.

$50 = 2 \times 5 \times 5$ Write factors from least to greatest.

Although the order of the factors may be different because you can start with different pairs of factors, every factor tree for 50 has the same **prime factorization**. You can also use **exponents** to write the prime factorization. $50 = 2 \times 5^2$



What is the prime factorization of each?

15. 40

16. 100

2.8 Using and Finding Percents

Finding a Percent of a Number

To find the percent of a number, you can use decimals or fractions. You first change the percent to a decimal or a fraction. Sometimes it is easier to change to a decimal representation and other times to a fractional one.

To find 25% of 60, you can use either the fraction method or the decimal method.

FINDING THE PERCENT OF A NUMBER: TWO METHODS

Find 25% of 60.

DECIMAL METHOD

- Change percent to decimal.

$$25\% = 0.25$$

- Multiply.

$$0.25 \times 60 = 15$$

So 25% of 60 = 15.

PERCENT METHOD

- Change percent to fraction.

$$25\% = \frac{25}{100}$$

- Reduce fraction to lowest terms.

$$\frac{25}{100} = \frac{1}{4}$$

- Multiply.

$$\frac{1}{4} \times \frac{60}{1} = \frac{60}{4} = 15$$

So 25% of 60 = 15.



Check It Out

Find the percent of each number.

1. 65% of 80

2. 85% of 500

3. 30% of 90

4. 75% of 420

Finding the Percent of a Number: Proportion Method

You can use proportions to help you find the percent of a number.

FINDING THE PERCENT OF A NUMBER: PROPORTION METHOD

Gwen works in a music store. She receives a commission of 10% on her sales. Last month she sold \$850 worth of CDs. What was her commission?

- Use a proportion to find the percent of a number.

$$P = \text{Part (of the base or total)} \quad \frac{P}{R} = \frac{B}{100}$$

$$B = \text{Base (total)} \quad R = \text{Rate (percentage)}$$

- Identify the given items before trying to find the unknown.

$$P \text{ is unknown, call } x. \quad R \text{ is } 10\%. \quad B \text{ is } \$850.$$

- Set up the proportion.

$$\frac{P}{R} = \frac{B}{100} \quad \frac{x}{10} = \frac{850}{100}$$

- Cross multiply.

$$100x = 8,500$$

- Divide both sides of the equation by the coefficient of x .

$$\frac{8,500}{100} = \frac{100x}{100} \quad 85 = x$$

Gwen received a commission of \$85.



Check It Out

Use a proportion to find the percent of each number.

5. 76% of 39

6. 14% of 85

7. 66% of 122

8. 55% of 300

Finding Percent and Base

Setting up and solving a proportion can help you find what percent a number is of a second number. Use the ratio $\frac{P}{B} = \frac{R}{100}$ where P = Part (of base), B = Base (total), and R = Rate (percentage).

FINDING THE PERCENT

What percent of 40 is 15?

- Set up a proportion, using this form.

$$\frac{\text{Part}}{\text{Base}} = \frac{\text{Percent}}{100}$$
$$\frac{15}{40} = \frac{n}{100}$$

(The number after the word *of* is the base.)

- Find the cross products of the proportion.

$$100 \times 15 = 40 \times n$$

- Find the products.

$$1,500 = 40n$$

- Divide both sides of the equation by the *coefficient* of n .

$$\frac{1,500}{40} = \frac{40n}{40} \quad \frac{1,500}{40} = n \quad 37.5\% = n$$

15 is 37.5% of 40.



Check It Out

Solve.

9. What percent of 240 is 80?
10. What percent of 64 is 288?
11. What percent of 2 is 8?
12. What percent of 55 is 33?

FINDING THE BASE

8 is 32% of what number?

- Set up a percent proportion using this form.

$$\frac{\text{Part}}{\text{Base}} = \frac{\text{Percent}}{100}$$
$$\frac{8}{n} = \frac{32}{100}$$

(The phrase *what number* after the word *of* is the base.)

- Find the cross products of the proportion.

$$8 \times 100 = 32 \times n$$

- Find the products.

$$800 = 32n$$

- Divide both sides of the equation by the coefficient of n .

$$\frac{800}{32} = \frac{32n}{32}$$
$$n = 25$$

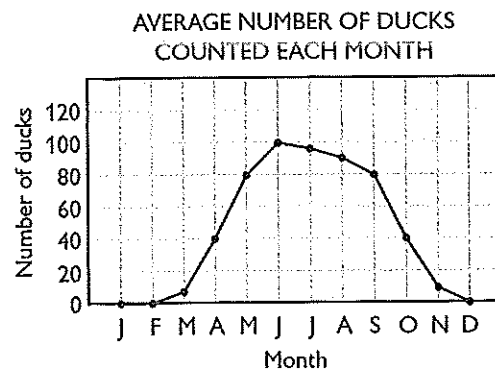
8 is 32% of 25.

Check It Out

13. 52 is 50% of what number?
14. 15 is 75% of what number?
15. 40 is 160% of what number?
16. 84 is 7% of what number?

Interpret a Line Graph

You know that a *line graph* can be used to show changes in data over time. The following line graph shows the average number of wood ducks to visit a duck pond over the year.



From the graph, you can see that there are fewer wood ducks in the area in winter months.



Check It Out

12. What was the greatest average number of ducks?
13. What month had the greatest average number of ducks?
14. Is this statement true or false: There were never more than 100 ducks in the pond.

Interpret and Create a Bar Graph

Another type of graph you can use to show data is called a *bar graph*. In this graph, either horizontal or vertical bars are used to show data. Consider the data showing the tallest buildings in the United States.

Building 1	346 m
Building 2	381 m
Building 3	343 m
Building 4	443 m
Building 5	419 m

You can make a bar graph to show these heights:

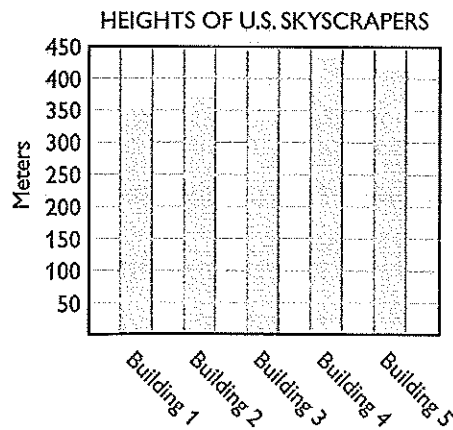
- Choose a vertical scale and decide what to place along the horizontal scale.

In this case, the vertical scale can show meters in increments of 50 m and the horizontal scale can show the buildings.

- Above each building, draw a bar of the appropriate height.
- Title the graph.

You can call it "Heights of U.S. Skyscrapers."

Your bar graph should look like this.



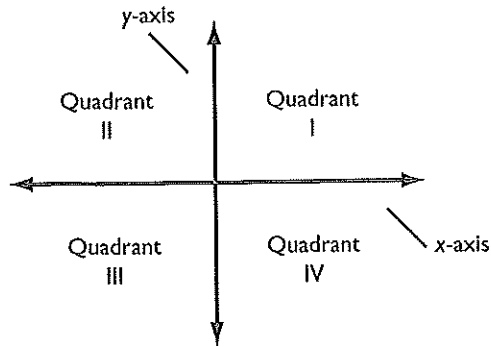
From the graph, you can see that the tallest building is Building 4.

6.7 Graphing on the Coordinate Plane

Axes and Quadrants

When you cross a horizontal (left to right) number line with a vertical (up and down) number line, the result is a two-dimensional coordinate plane.

The number lines are called axes. The horizontal number line is the **x-axis** and the vertical number line is the **y-axis**. The plane is divided into four regions, called **quadrants**. Each quadrant is named by a roman numeral, as shown in the diagram.



Check It Out

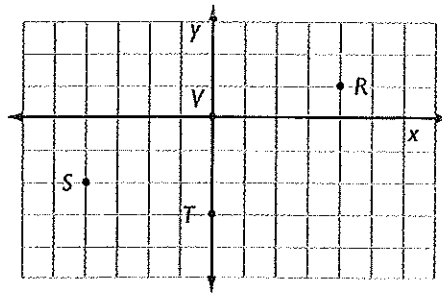
Fill in the blanks.

1. The horizontal number line is called the _____.
2. The lower left region of the coordinate plane is called _____.
3. The upper right region of the coordinate plane is called _____.
4. The vertical number line is called the _____.

Writing an Ordered Pair

Any location on the coordinate plane can be represented by a point. The location of any point is given in relation to where the two axes intersect, called the origin.

Two numbers are required to identify the location of a point. The x -coordinate tells you how far to the left or right of the origin the point lies. The y -coordinate tells you how far up or down from the origin the point lies. Together, the x -coordinate and y -coordinate form an ordered pair, (x, y) .



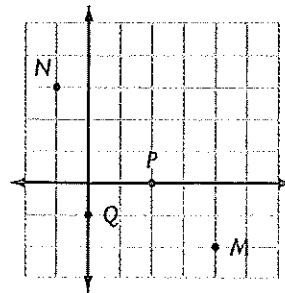
Since point R is 4 units to the right of the origin and 1 unit up, its ordered pair is $(4, 1)$. Point S is 4 units to the left of the origin and 2 units down, so its ordered pair is $(-4, -2)$. Point T is 0 units to the left or right of the origin and 3 units down, so its ordered pair is $(0, -3)$. Point V is 0 units to the left or right of the origin and 0 units up or down. Point V is the origin, and its ordered pair is $(0, 0)$.



Check It Out

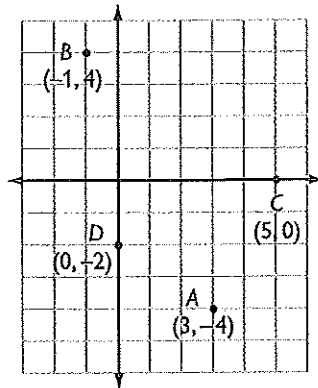
Give the ordered pair for each point.

5. M
6. N
7. P
8. Q



Locating Points on the Coordinate Plane

To locate point $A(3, -4)$ from the origin, move 3 units to the right and 4 units down. Point A lies in Quadrant IV. To locate point $B(-1, 4)$ from the origin, move 1 unit to the left and 4 units up. Point B lies in Quadrant II. Point $C(5, 0)$ is, from the origin, 5 units to the right and 0 units up or down. Point C lies on the x -axis. Point $D(0, -2)$ is, from the origin, 0 units to the left or right and 2 units down. Point D lies on the y -axis.



Check It Out

Draw each point on the coordinate plane and tell where it lies.

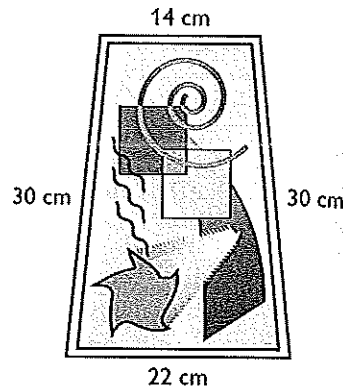
9. $H(2, -3)$
10. $J(-3, 2)$
11. $K(2, 1)$
12. $L(0, 4)$

7.4 Perimeter

Perimeter of a Polygon

Dolores is planning to put a frame around a painting. To determine how much framing she needs, she must calculate the perimeter of, or *distance around*, the painting.

The perimeter of any polygon is the sum of the lengths of the sides of the polygon.



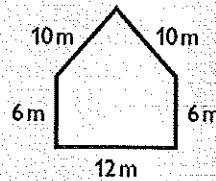
$$P = 30 \text{ cm} + 30 \text{ cm} + 14 \text{ cm} + 22 \text{ cm} = 96 \text{ cm}$$

The perimeter of the painting is 96 cm. Dolores will need 96 cm of framing.

FINDING THE PERIMETER OF A POLYGON

To find the perimeter of any polygon, add up the lengths of all its sides.

Find the perimeter.



$$P = 6 \text{ m} + 10 \text{ m} + 10 \text{ m} + 6 \text{ m} + 12 \text{ m} = 44 \text{ m}$$

The perimeter of this pentagon is 44 m.

Regular Polygon Perimeters

The sides of a regular polygon are all the same length. If you know the perimeter of a regular polygon, you can find the length of each side.

To find the length of each side of a regular hexagon with a perimeter of 15 cm, let x = length of a side.

$$15 \text{ cm} = 6x \qquad 2.5 \text{ cm} = x$$

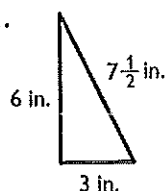
Each side is 2.5 cm long.



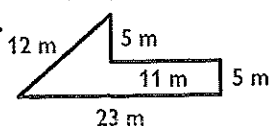
Check It Out

Find the perimeter of each polygon.

1.



2.



Find the length of each side.

3. a regular octagon with a perimeter of 32 ft
4. an equilateral triangle with a perimeter of 63 m

Perimeter of a Rectangle

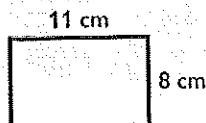
Opposite sides of a rectangle are equal. So to find the perimeter of a rectangle, you only need to know its length and width.

FINDING THE PERIMETER OF A RECTANGLE

For a rectangle with length, l , and width, w , the perimeter, P , can be found with the formula $P = 2l + 2w$.

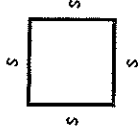
Find the perimeter of a rectangle with a length of 11 cm and a width of 8 cm.

$$\begin{aligned} P &= 2l + 2w \\ &= (2 \times 11) + (2 \times 8) \\ &= 22 + 16 = 38 \text{ cm} \end{aligned}$$



The perimeter is 38 centimeters.

A square is a rectangle whose length and width are equal. So the formula for finding the perimeter of a square, whose sides measure s , is $P = 4 \times s$ or $P = 4s$.



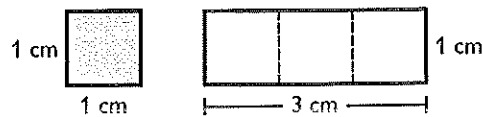
Check It Out

Find the perimeter.

5. rectangle with length 10 ft and width 5 ft
6. square with sides of 9 in

What Is Area?

Area measures the size of a surface. Your desktop is a surface with area, and so is the state of Florida. Instead of measuring with units of length, such as inches, centimeters, feet, and kilometers, you measure area in square units, such as square inches (in.^2) and square centimeters (cm^2).

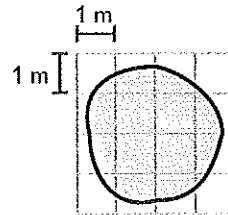


This square has an area of one square centimeter. It takes exactly three of these squares to cover this rectangle, which tells you that the area of the rectangle is three square centimeters, or 3 cm^2 .

Estimating Area

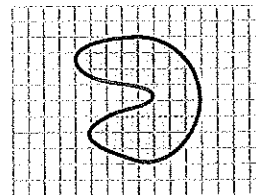
When an exact answer is not needed or is hard to find, you can estimate the area of a surface.

In the shaded figure to the right, four squares are completely shaded, and so you know that the area is greater than 4 m^2 . The square around the shape covers 16 m^2 , and obviously the shaded area is less than that. So to estimate the area of the shaded figure, you can say that it is greater than 4 m^2 but less than 16 m^2 .



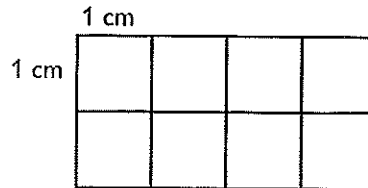
Check It Out

1. Estimate the area of the shaded region. Each square represents 1 m^2 .



Area of a Rectangle

By counting squares you can find the area of this rectangle.



There are 8 squares and each is a square centimeter. So the area of this rectangle is 8 cm^2 .

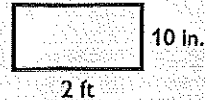
You can also use the formula for finding the area of a rectangle: $A = l \times w$. The length of the rectangle above is 4 cm and the width is 2 cm. Using the formula, you find that

$$\begin{aligned} A &= 4 \text{ cm} \times 2 \text{ cm} \\ &= 8 \text{ cm}^2 \end{aligned}$$

FINDING THE AREA OF A RECTANGLE

Find the area of this rectangle.

- The length and the width must be in the same units.




$$2 \text{ ft} = 24 \text{ in. So } l = 24 \text{ in. and } w = 10 \text{ in.}$$

- Use the formula for the area of a rectangle.

$$\begin{aligned} A &= l \times w \\ &= 24 \text{ in.} \times 10 \text{ in.} \\ &= 240 \text{ in.}^2 \end{aligned}$$

The area of the rectangle is 240 in.^2 .

If the rectangle is a square, the length and the width are the same. So for a square whose sides measure s units, you can use the formula $A = s \times s$, or $A = s^2$.

 **Check It Out**

2. Find the area of a rectangle with a length of 3 ft and a width of 9 in.
3. Find the area of a square whose sides measure 13 m.

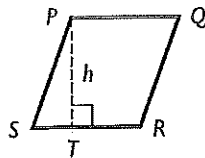
Area of a Parallelogram

To find the area of a parallelogram, you multiply the base by the height.

Area = base \times height

$$A = b \times h$$

or $A = bh$




The height of a parallelogram is always **perpendicular** to the base. So in parallelogram $PQRS$, the height, h , is equal to \overline{PT} , not \overline{PS} . The base, b , is equal to \overline{SR} .

FINDING THE AREA OF A PARALLELOGRAM

Find the area of a parallelogram with a base of 9 in. and a height of 5 in.

$$\begin{aligned} A &= b \times h \\ &= 9 \text{ in.} \times 5 \text{ in.} \\ &= 45 \text{ in.}^2 \end{aligned}$$

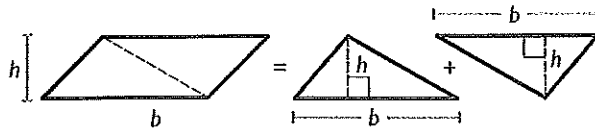
The area of the parallelogram is 45 in.^2 or 45 sq in.

 **Check It Out**

4. Find the area of a parallelogram with a base of 6 cm and a height of 8 cm.
5. Find the height of a parallelogram that has a base of 12 ft and an area of 132 ft^2 .

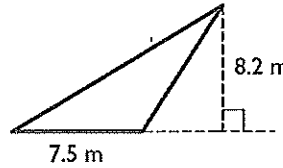
Area of a Triangle

If you were to cut a parallelogram along a diagonal, you would have two triangles with equal bases, b , and the same height, h .



A triangle has half the area of a parallelogram with the same base and height. The area of a triangle equals $\frac{1}{2}$ the base times the height, and so the formula is $A = \frac{1}{2} \times b \times h$, or $A = \frac{1}{2}bh$.

$$\begin{aligned} A &= \frac{1}{2} \times b \times h \\ A &= \frac{1}{2} \times 7.5 \times 8.2 \\ &= 0.5 \times 7.5 \times 8.2 \\ &= 30.75 \text{ m}^2 \end{aligned}$$



The area of the triangle is 30.75 m^2 .

FINDING THE AREA OF A TRIANGLE

Find the area of $\triangle PQR$. Note that in a right triangle, the two legs serve as a height and a base.

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 4 \times 6 \\ &= 0.5 \times 4 \times 6 \\ &= 12 \text{ ft}^2 \end{aligned}$$

The area of the triangle is 12 ft^2 .

For a review of *right triangles*, see page 378.



Check It Out

6. Find the area of a triangle with a base of 15 cm and a height of 10 cm.
7. Find the area of a right triangle whose sides measure 10 in., 24 in., and 26 in.

